A Comment on the Extractions of V_{ub} from Radiative Decays

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Abstract

We present a model independent closed form expression for $|V_{ub}|^2/|V_{tb}V_{ts}^*|^2$, which includes the resummation of large endpoint logarithms as well as the interference effects from the operators O_2 and O_8. We demonstrate that the method to extract $|V_{ub}|$ presented by the authors in hep-ph/9909404, and modified in this letter to include interference effects, is not just a refinement of the method introduced in hep-ph/9312311. We also discuss the model dependence of the latter proposal. Furthermore, we show that the resummation is not negligible and that the Landau pole does not introduce any significant uncertainties.

Testing the Standard Model in the Cabbibo-Kobayashi-Maskawa sector has been hindered by the relatively large uncertainties in the matrix element V_{ub}. The absolute value of this matrix element has been extracted by the study of inclusive charmless B decays, with large uncertainties from model dependence. There really is no way to define a theoretical error in this extraction, since the calculations are not based on a controlled expansion. The model dependence is introduced as a consequence of the need to make a cut on the electron energy spectrum near the endpoint to eliminate the large background from charmed decays. This probing of the endpoint region makes the cut rate sensitive to the Fermi motion of the heavy quark inside the hadron. In the past, one has needed to use models for the Fermi motion leading to the aforementioned uncontrolled errors. It is now well known that it is possible to avoid the model dependence by using the data from radiative decays to eliminate the dependence of the Fermi motion.

In this note we will discuss two proposals for implementing this idea. One, introduced by Neubert [1], and the other by the authors [2] based on ideas of Korchemsky and Sterman [3]. We will show that the results in [2] are not just a refinement of Neubert's proposal, which is model dependent, whereas the results of [2] are not. We will further demonstrate that there is a well defined prescription to handle the Landau singularity which is unambiguous. Finally, we will show that, when using the present experimental cut, the effect of resummation is not negligible.

Let us first review Neubert's proposal [1], as recently updated to include interference effects in [4]. At *tree* level the decay rate near the endpoint may be written as [1,5]

$$\frac{d\Gamma}{dx} = \frac{G_F^2 |V_{ub}|^2 m_b^5}{96\pi^3} \left[F(x)\theta(1-x) + F(1)S(x) \right], \tag{1}$$

where $x = 2E_e/m_b$, and $F(x) \approx F(1)$ near the endpoint. This result follows from taking the imaginary part of the tree level current-current correlator. At leading order in Λ/m_b, we may write

$$\theta(1-x) + S(x) = \langle B|\theta(1-x+in\cdot\hat{D})|B\rangle, \tag{2}$$

where n_μ is a light-like vector satisfying $n \cdot v = 1$, and $\hat{D}^\mu = D^\mu/m_b$. The photon spectrum in radiative decay may similarly be written, also at tree level, as

$$\frac{d\Gamma^\gamma}{dx} = \frac{G_F^2 \, \alpha \, m_b^5 \, C_7^2}{32\pi^4} |V_{tb}V_{ts}^*|^2 \langle B|\delta(1 - x + in \cdot \hat{D})|B\rangle. \tag{3}$$

Then using the relation

$$\int_x^\infty dx'(x' - x)\langle B|\delta(1 - x' + in \cdot \hat{D})|B\rangle = \int_x^\infty dx' \langle B|\theta(1 - x' + in \cdot \hat{D})|B\rangle, \tag{4}$$

one can write

$$\left|\frac{V_{ub}}{V_{tb}V_{ts}^*}\right|^2 = \frac{3\alpha}{\pi}|C_7|^2\frac{\Gamma_u(E_c)}{\Gamma_s(E_c)} + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda/m_b), \tag{5}$$

where $\Gamma_i(E_c)$ is the cut integrated rate. To take into account the perturbative corrections, the author of [1] adds a correction factor η_{QCD}. In [1] η_{QCD} is given by

$$\eta_{\mathrm{QCD}} = 1 + \frac{2\alpha_s}{9\pi}\left(5\log(r) + \pi^2 - \frac{35}{4}\right). \tag{6}$$

The quantity r is unknown, and depends upon the non-perturbative structure function. This structure function dependence arises because it is not truly possible to cancel off the soft effects in this way, once the radiative corrections are included, because these two effects are convoluted.

However, Neubert derived the following bound

$$-\log(r) > -\log\left(1 - x_B^c\right). \tag{7}$$

While this bound is helpful, it does not really tell us much about the relative size of the model dependence. Varying r within its allowed range can significantly change the radiative corrections. In Fig. 1 we plot the parameter K_{pert} defined in Eq. (3) of Ref. [4], which updates η_{QCD} by including interference effects, as a function of r. It is clear that K_{pert} is quite sensitive to the value of r and, unfortunately, a priori we have no idea what value of r to choose.
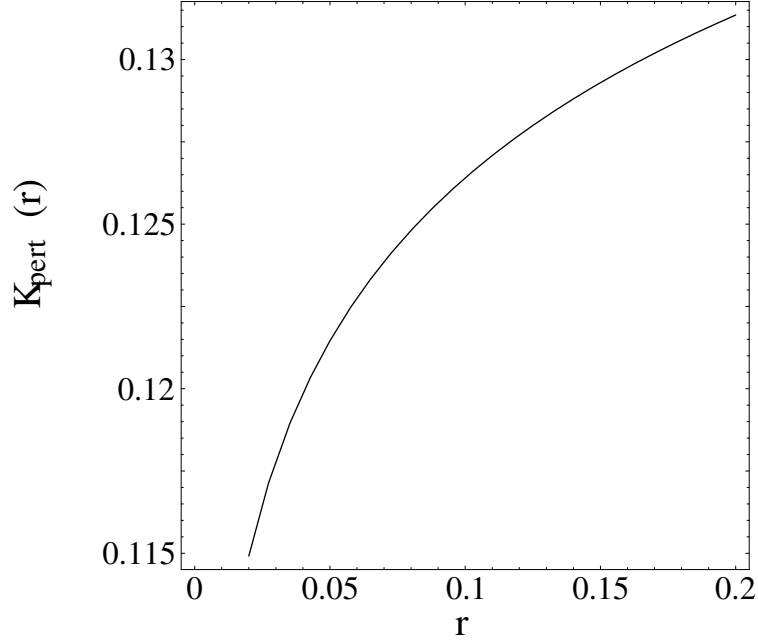
FIG. 1. $K_{\rm pert}$ as a function of the non-perturbative parameter r in the range $0.02 < r < 0.2$.

The proposal of Ref. [2], on the other hand, has no model dependence. The calculations, based on the factorization shown by Korchemsky and Sterman [3] and the results of [6], lead to

$$
\frac{|V_{ub}|^2}{|V_{ts}^* V_{tb}|^2} = \frac{3\,\alpha\,|C_7(m_b)|^2}{\pi} \int_{x_B^c}^1 dx_B \frac{d\Gamma}{dx_B}
$$
$$
\times \left\{ \int_{x_B^c}^1 dx_B \int_{x_B}^1 du_B\, u_B^2\, \frac{d\Gamma^\gamma}{du_B}\, K\left[x_B; \frac{4}{3\pi\beta_0} \log(1 - \alpha_s \beta_0\, l_{x_B/u_B}) \right] \right\}^{-1}, \qquad (8)
$$

where the expression for K can be found in [2] and $l_{x/u} = -\log[-\log(x/u)]$. x_B^c is the larger of the two energy cuts for the electron energy spectrum of $B \to X_u e\nu$ and the photon energy spectrum of $B \to X_s \gamma$. In addition to including the full $\mathcal{O}(\alpha_s)$ corrections, this result also includes a summation of the next-to-leading Sudakov logarithms $(\log(1 - x_B^c))$ which become large as x_B^c approaches one. This result may be re-written as

$$
\frac{|V_{ub}|^2}{|V_{ts}^* V_{tb}|^2} = \frac{3\,\alpha\,C_7(m_b)^2}{\pi} \int_{x_B^c}^1 dx_B \frac{d\Gamma}{dx_B} \times \left\{ \int_{x_B^c}^1 du_B W[u_B, x_B^c] \frac{d\Gamma^\gamma}{du_B} \right\}^{-1}, \qquad (9)
$$
$$
W[u_B, x_B^c] = u_B^2 \int_{x_B^c}^{u_B} dx_B\, K\left[x_B; \frac{4}{3\pi\beta_0} \log(1 - \alpha_s \beta_0\, l_{x_B/u_B}) \right], \qquad (10)
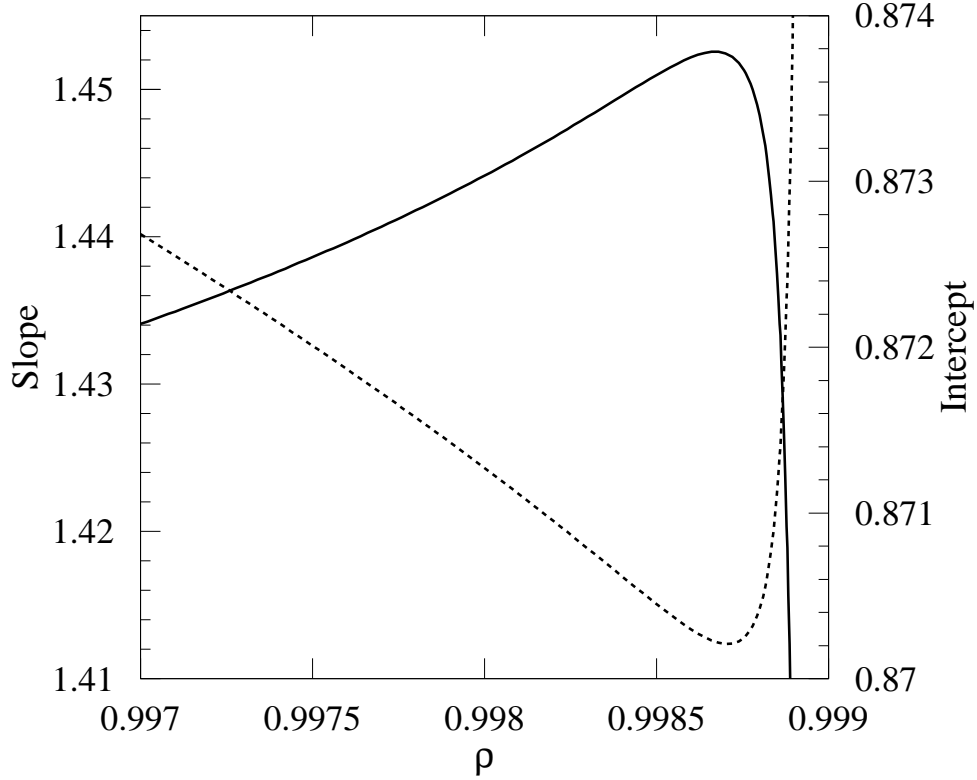$$

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FIG. 2. *The slope (solid line) and x-axis intercept (dotted line) of the weight function as a function of ρ for $x_B^c = 0.87$ and $\alpha_s = 0.21$.*

where $W[u_B, x_B^c]$ is a weighting function which is approximately linear.

Next we would like to address the issue of the Landau pole. The argument of K diverges when $1 - \alpha_s \beta_0 \, l_{x_B/u_B} = 0$. In the denominator of Eq. (9) the integration region is a triangular region bounded by $x_B^c \leq x_B \leq u_B \leq 1$. The Landau pole is located at $(x_B/u_B)_{max} = 1 - \exp[-1/(\alpha_s \beta_0)] \approx 0.999$. One way to avoid the pole is to integrate over the region $x_B \leq \rho \, u_B$, where $\rho \lesssim 0.999$. Since the physical radiative rate is a smooth function, the area we cut off from the integration region should not incur substantial error in the extraction of $|V_{ub}|$. When cutting the integration region as described here, the weight function remains approximately linear.

However, an important question in practice is how close we can get to the Landau pole

region [7]. This question arises because, as we get very close to the Landau pole, the perturbative resummation breaks down. Ideally, we would like to cut off as little integration region as possible while still leaving a well-behaved perturbative resummation. After the introduction of ρ, Eqs. (9) and (10) become

$$\frac{|V_{ub}|^2}{|V_{ts}^*V_{tb}|^2} = \frac{3\,\alpha\,|C_7(m_b)|^2}{\pi} \int_{x_B^c}^1 dx_B \frac{d\Gamma}{dx_B} \times \left\{ \int_{x_B^c/\rho}^1 du_B \frac{d\Gamma^\gamma}{du_B} W[u_B, x_B^c, \rho] \right\}^{-1}, \qquad (11)$$

$$W[u_B, x_B^c, \rho] = u_B^2 \int_{x_B^c}^{\rho\,u_B} dx_B\, K\left[x_B; \frac{4}{3\pi\beta_0} \log(1 - \alpha_s\beta_0\,l_{x_B/u_B}) \right]. \qquad (12)$$

To determine the optimal value of ρ, we plot in Fig. 2 the slope and x-axis intercept of the weight function, Eq. (12), for various values of ρ. It is clear that, as ρ varies from 0.97 to 0.998, W converges to an asymptote. However, for $\rho \sim 0.9988$, the perturbative expansion breaks down, as is evident from the abrupt change in the behavior of the curves. The weight function changes abruptly, which signals the breakdown of perturbative resummation.

As ρ gets smaller, we are cutting off more of the integration region, which results in a weight function with different intercepts and slopes, as shown in Fig. 2. However, we have to bear in mind that this is an approximation scheme. The more we cut off the integration region, the worse an approximation it is. Therefore, it is not at all surprising that different values of ρ yields different values of $|V_{ub}|$ when using Eq. (11). Ideally we would like ρ to be as close to unity as possible, in order to have a good approximation, while maintaining a controlled resummation. It is clear from Fig. 2 that the true weight function is approached asymptotically. When performing the analysis experimentally, we can either use $\rho = 0.9987$, or try to extrapolate W all the way up to $\rho = 1$. The difference should be well within the theoretical error.

Another prescription for avoiding the Landau pole is to expand the second argument of K in Eq. (10) as a power series in α_s,

$$K\left[x, \frac{4}{3\pi\beta_0} \log(1 - \alpha_s\beta_0 l_{x_B/u_B}) \right] = K\left[x, -\frac{4}{3\pi}\left(\alpha_s l_{x_B/u_B} + \frac{1}{2}\alpha_s^2\beta_0 l_{x_B/u_B}^2 + \cdots \right) \right]. \qquad (13)$$
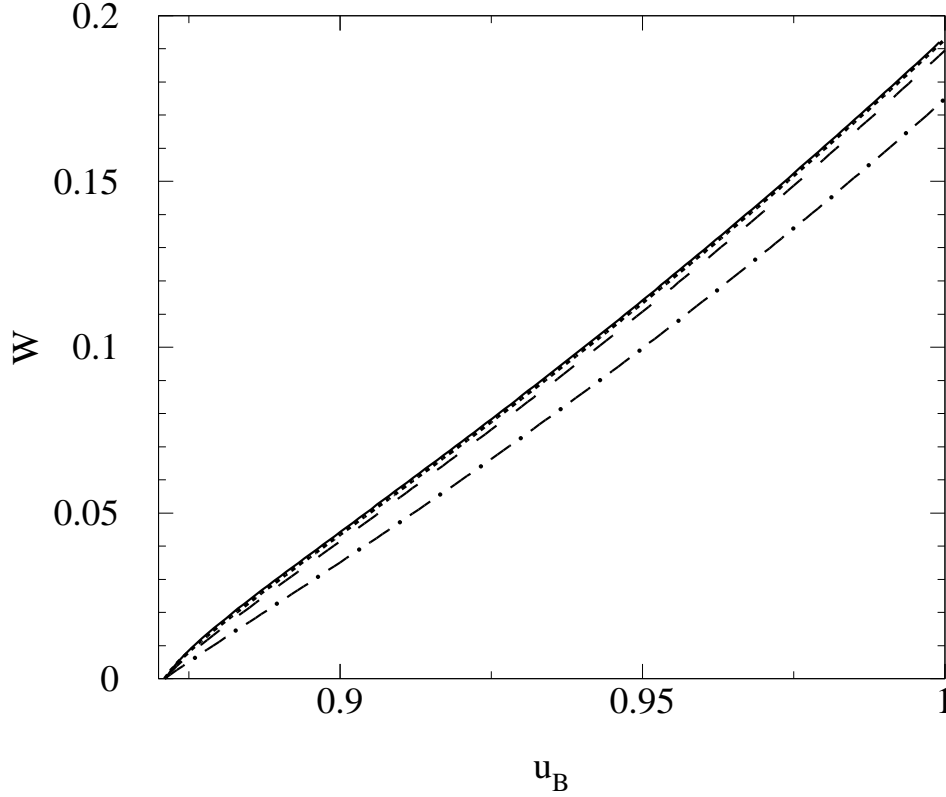
FIG. 3. *Weight function obtained by expanding the argument of K in Eq. (10) to different powers of α_s, using $x_B^c = 0.87$ and $\alpha_s = 0.21$. The dot-dashed line is expanding to order α_s, the dashed line to order α_s^3 and the dotted line to α_s^5. The weight function is quickly converging to the solid line, which is the weight function from Eq. (11) using $\rho = 0.9987$.*

This corresponds to expanding g_{sl} of Ref. [2] in the exponent.[1] We can check the convergence of this prescription by expanding to different orders in α_s. In Fig. 3, we expand the argument of K to orders α_s (dot-dashed line), α_s^3 (dashed line) and α_s^5 (dotted line). We also show

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[1]Note that expanding in the exponent is not equivalent to expanding in α_s. Indeed an expansion in α_s (i.e., expanding K as a series in α_s) leads to a very poorly behaved series. The beauty of the resummation is that the series is reorganized in such a way that the expansion in the exponent is well behaved [8].

(solid line) the weight function from Eq. (12) using $\rho = 0.9987$. It is clear that the expansion is quickly converging, and it is converging to the weight function using the other prescription. It is therefore evident that there is an unambiguous choice of weighting function which can be used, with negligible error introduced.

Now we would like to discuss the effect of resummation.[2] Since the weight function in Eq. (10) is approximately linear, we plot in Fig. 4 the slope of the weight function with the fully resummed result versus the slope without resumming the Sudakov logarithms, with the choice of $\rho = 0.9987$. We see that the resummation has roughly a 10% effect on the slope of the weight function, for the current experimental cut on the electron energy spectrum, $E_{cut} = 2.3$ GeV or $x_B^c = 0.87$. In our original paper, Ref. [2], we proposed to use $\rho = 0.99$. However, at that time we did not fully investigate the sensitivity due to the Landau pole. Had we used the choice $\rho = 0.99$ in Fig. 4, we would have found that the resummation has a very small effect. This is because we would have cut off a region where the Sudakov logarithms are important. Now it should be clear that, when the optimal value of ρ is used, the resummation does have a non-negligible effect.

Finally, it was correctly pointed out in [4] that we mistakenly neglected the contribution from interference terms which can be large when studying the integrated radiative decay rate. At leading order, the only operator that is important is O_7, the electromagnetic penguin operator. At order α_s in the decay rate, O_7 interferes with O_2 and O_8 [11]. The contribution from other operators are small and can be neglected. The contribution from $O_2 O_7$ and $O_7 O_8$ terms are also suppressed by exponentiated Sudakov logarithms, and can be included trivially in our formula by changing the overall factor in Eq. (9) or Eq. (11) to[3]

$$\frac{|V_{ub}|^2}{|V_{ts}^* V_{tb}|^2} = \frac{3\,\alpha\,C_7^{(0)}(m_b)^2}{\pi}(1 + H_{mix}^\gamma) \int_{x_B^c}^1 dx_B \frac{d\Gamma}{dx_B} \times \left\{ \int_{x_B^c}^1 du_B W(u_B) \frac{d\Gamma^\gamma}{du_B} \right\}^{-1}, \qquad (14)$$

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[2] For other work on resumming endpoint logs see [9,10].

[3] When using the hadronic mass spectrum to extract V_{ub} [12], we should take into account the interference effect in a similar fashion.
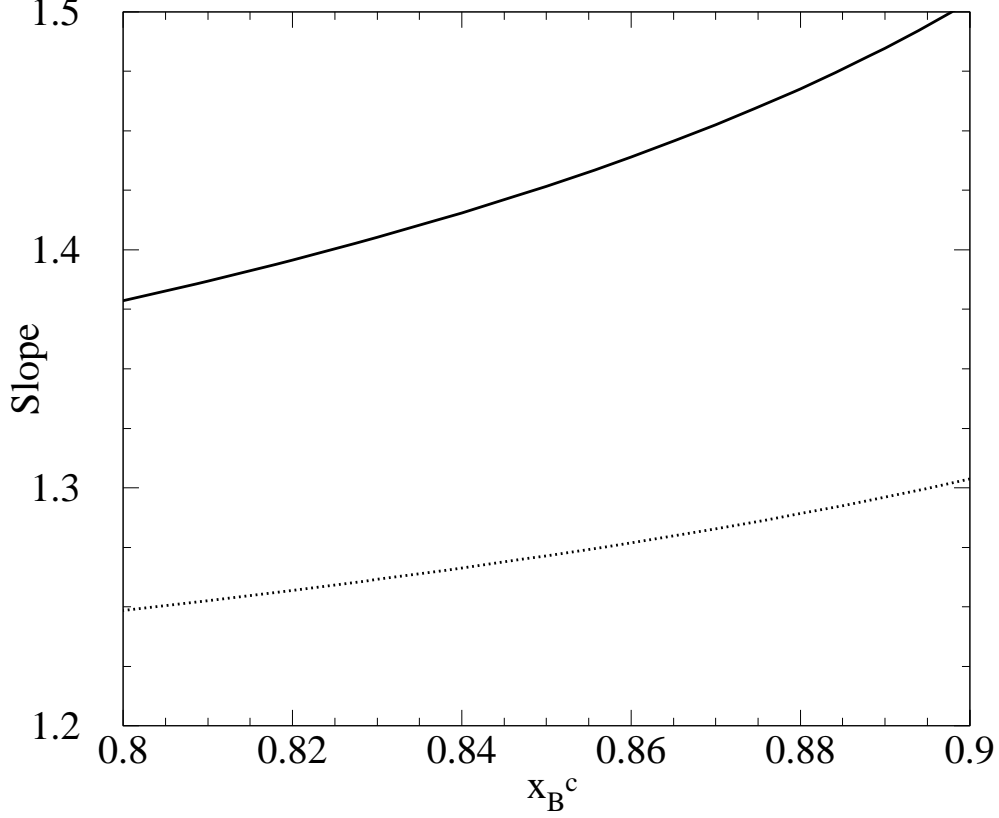
FIG. 4. *The slope of the weight function as a function of the cut showing the effects of resummation. The dotted line is the slope without resumming the Sudakov logarithms.*

where

$$H^\gamma_{mix} = \frac{\alpha_s(m_b)}{2\pi C_7^{(0)}} \left[C_7^{(1)} + C_2^{(0)} \Re(r_2) + C_8^{(0)} \left(\frac{44}{9} - \frac{8\pi^2}{27} \right) \right]. \tag{15}$$

In Eq. (15), all the Wilson coefficients, evaluated at m_b, are "effective" as defined in [13], and $\Re(r_2) \approx -4.092 + 12.78(m_c/m_b - 0.29)$ [11]. The numerical values of the Wilson coefficients are [14]: $C_2^{(0)}(m_b) \approx 1.11$, $C_7^{(0)}(m_b) \approx -0.31$, $C_7^{(1)}(m_b) \approx 0.48$, and $C_8^{(0)}(m_b) \approx -0.15$. With this expression in hand we believe it to be relatively straightforward to extract $|V_{ub}|^2$ with theoretical errors on the order of Λ/m_b.

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